On a proposed new test of Heisenberg's principle

1980 J. Phys. A: Math. Gen. 13 877

(http://iopscience.iop.org/0305-4470/13/3/020)

J. Phys. A: Math. Gen. **13** (1980) 877–880. Printed in Great Britain

On a proposed new test of Heisenberg's principle

M C Robinson
Departmento de Fisica, Universidad de Oriente, Cumaná, Venezuela

Received 3 October 1978, in final form 7 August 1979

Abstract. The design of a technically feasible experiment is discussed that would distinguish between Heisenberg's uncertainty principle and de Broglie's relation, $(\Delta x)_i(\Delta p)_f \geqslant \hbar/2$ where $(\Delta x)_i$ is the initial localisation of the particle and $(\Delta p)_f$ is the final scatter in the momentum of the free particle.

We have previously proposed a thought experiment (Robinson 1969a) which, in principle, could lead to a violation of Heisenberg's uncertainty principle, $\Delta x\ \Delta p \geqslant \hbar/2$. The experiment consisted of two detectors, D_1 and D_2, at x_1 and x_2 respectively, separated by a velocity selector. The particles enter the first detector one at a time. The wavefunction of the particle (or of the ensemble of particles in some interpretations of quantum mechanics) thus becomes localised in real space and thereby smeared out in momentum space. Occasionally one of the particles is scattered at D_1 with a velocity parallel to the x axis and in the narrow range $(v_s - \delta v_s, v_s + \delta v_s)$ enabling it to pass through the selector. Thus the wavefunction is now localised in momentum space with an uncertainty $\Delta p = m\Delta v_s < m\delta v_s$, the inequality following from the fact that the RMS deviation in the velocity, Δv_s, is less than the maximum deviation, δv_s. Therefore, according to Heisenberg's principle, the position of the particle is uncertain (or undetermined or smeared out in real space) by an amount characterised by

$$\Delta x \geqslant \hbar/(2\Delta p) > \hbar/(2m\delta v_s).$$

The wavefunction will be of the form $\psi(x, t) = f(x, t)\exp(ikx)$, where $k = mv_s/\hbar$, and $f(x, t)$ is approximately constant in magnitude in an interval equal to $2\Delta x$ initially. This wave packet will move through the selector with a velocity v_s.

According to the usual interpretation of quantum mechanics, the detector D_2 will register the presence of the particle at any instant during the time it takes for the wave packet to pass the point x_2. Therefore, if these measurements are repeated several times there will be fluctuations in the times of flight with an RMS value

$$\Delta t = \Delta x/v_s.$$

Thus, in general, the time of flight velocity will be

$$v_t = (x_2 - x_1)(t_2 - t_1)^{-1} \neq v_s$$

but the average value $\langle v_t \rangle = v_s$. Here t_1 and t_2 refer to the times of detection of the particle at x_1 and x_2, respectively.

If, however, $v_t = v_s$, for every repetition of the experiment, we can conclude that the position of the particle is given by

$$x(t) = x_1 + v_s(t - t_1) = x_2 - v_s(t_2 - t)$$

during any instant of time, t, in the interval (t_1, t_2). The uncertainty in the position is given by the smaller of the quantities $\delta v_s(t - t_1)$ and $\delta v_s(t_2 - t)$, assuming that the experiment was so designed that Δx_1 and $v_s \Delta t_1 \ll \delta v_s(t - t_1)$, and Δx_2 and $v_s \Delta t_2 \ll \delta v_s(t_2 - t)$. In this case, $\Delta x < \delta v_s(t_2 - t_1)$. Since in principle $\delta v_s(t_2 - t_1)$ can be made arbitrarily small, we can make a series of measurements such that

$$\Delta x\ \Delta p < m(\delta v_s)^2(t_2 - t_1) < \hbar/2. \tag{1}$$

Not only would this represent a violation of Heisenberg's principle, but also of the statistical or probabilistic interpretation of ψ since all the particles that passed through the selector would be situated in the centre of the wave packet.

The result $v_t = v_s$ would however be in agreement with the pilot wave (or double solution) interpretation of quantum mechanics (de Broglie 1927, 1964; Bohm 1952a, b) where ψ represents a quantum force field with the properties that $|\psi|^2$ usually, but not necessarily always, equals a probability density in real space. On the other hand, in general the expectation value of a dynamical variable, $\langle F \rangle \neq \langle \psi | F_{\mathrm{op}} | \psi \rangle$ where F_{op} is the associated operator; in fact, dynamical variables are represented as in classical mechanics and not by associated operators. A consequence of this interpretation is a restricted form of the uncertainty principle due to de Broglie (1969) (see also Andrade e Silva 1967) according to which $(\Delta x)_{\mathrm{i}} (\Delta p)_{\mathrm{f}} \geqslant \hbar/2$ where $(\Delta x)_{\mathrm{i}}$ represents the initial localisation and $(\Delta p)_{\mathrm{f}}$ is the final scatter in the momentum of a free particle. It is assumed that the particle has a definite position and momentum at every instant of time, and places no prohibition upon the determination of the simultaneous values of these quantities.

In our analysis of the practicality of the above experiment, we found that it would be extremely difficult to realise. Since, however, an error was made in the analysis, we shall reconsider this question below. It will be shown that by changing the shape of the selector from linear to circular, the proposed experiment becomes feasible.

D_1 could consist of a very thin film of a metal with a small work function deposited on a transparent insulator such as aluminia. The detector, D_1, is illuminated periodically, say once a second, with very short pulses of monochromatic radiation with a frequency chosen so that the maximum velocity of the photoelectrons would only be slightly greater than v_s. The second detector, D_2, could be a photomultiplier. In the linear configuration of the velocity selector, it would consist of two parallel plates of a charged capacitor separated by a small distance, d, and extending from D_1 at x_1 to D_2 at x_2. The plates are taken to be in the xz plane so that the constant electric field, E, will be parallel to the y axis. A constant magnetic field, B, is parallel to the z axis.

If the velocity of the electron is $v_s \pm \delta v_s$, then the particle will have a circular motion in the xy plane superposed upon the average velocity, $v_s = E/B$, in the x direction. The angular velocity will be $\omega = qB/m$ and the radius of the orbit will be equal to $\delta v_s/\omega$. For the particle to pass through the selector without being absorbed in one of the plates, this radius must be less than half the separation of the plates, d, so that the maximum value of $\delta v_s = qBd/(2m)$. Hence

$$\Delta x > \hbar/(qBd).$$

If $d = 0{\cdot}1$ mm, then for a value of $\Delta x > 1$ cm, it is necessary that $B = 0{\cdot}7 \times 10^{-5}$ G. If the energy of the electrons is $0{\cdot}01$ eV, $v_s = 6 \times 10^4\ \mathrm{ms}^{-1}$, so that according to the usual interpretation, the RMS fluctuations in the time of flight will be

$$\Delta t = \Delta x / v_s > 1{\cdot}7 \times 10^{-7}\ \mathrm{s}$$

with about a third of the time-of-flight measurements showing a deviation greater than this. If $x_2 - x_1 = 1$ m, then the average time of flight will be $1{\cdot}7 \times 10^{-5}$ s.

Measuring such time fluctuations is clearly feasible. There will be, of course, a second source of fluctuations due to the uncertainty, δv_s, in the selector velocity. Substituting the design figures used above gives $\delta v_s = 5{\cdot}8 \times 10^{-3}\ \mathrm{ms}^{-1}$ which would result in a maximum fluctuation in the time of flight of about $0{\cdot}1$ ppm or $1{\cdot}7 \times 10^{-12}$ s. This value, obviously, would present no problem.

If, however, our experiment shows us that $v_t = v_s$, we shall have, upon substituting the above values for δv_s and $t_2 - t_1$ in equation (1),

$$\Delta x\ \Delta p < 5 \times 10^{-40}\ \mathrm{J\,s} \ll \hbar/2$$

which represents a clear violation of Heisenberg's uncertainty principle.

The main difficulty in realising this experiment would be in maintaining E and B constant to better than 1 ppm over a distance of 1 m. This problem becomes considerably less difficult if the plates are bent in a circular arc of radius, r, and only a magnetic field, B, is applied, i.e. $E = 0$. Such a selector could be accurately centred in the centre of a long superconducting magnet carefully shielded against any external field.

In this circular configuration, $v_s = qBr/m$ and $\delta v_s = qB\delta r/m$, where δr is the separation between the plates. If the overall length of the selector is kept constant at 1 m and we set $r = 1$ m, $\delta r = 0{\cdot}01$ mm and $B = 3{\cdot}3 \times 10^{-4}$ G, we obtain $v_s = 5{\cdot}8 \times 10^3\ \mathrm{ms}^{-1}$, $\delta v_s = 5{\cdot}8 \times 10^{-2}\ \mathrm{ms}^{-1}$ and $t_2 - t_1 = 1{\cdot}7 \times 10^{-4}$ s. Denoting the uncertainty in the position along the length of the selector by $\Delta\xi$ and the uncertainty of the tangential momentum by Δp_ξ, we have from Heisenberg's principle $\Delta\xi \geqslant \hbar/(2\Delta p_\xi) > \hbar/(2m\delta v_s) = 1{\cdot}0$ mm. The expected RMS fluctuation in the time of flight is given by $\Delta\xi/v_s > 1{\cdot}7 \times 10^{-7}$ s. If, however, $v_t = v_s$, then the uncertainty in the position, $\Delta\xi < \delta v_s(t_2 - t_1)$ and

$$\Delta\xi\ \Delta p_\xi < m(\delta v_s)^2(t_2 - t_1) = 5{\cdot}3 \times 10^{-37}\ \mathrm{J\,s} < \hbar/2,$$

once more violating Heisenberg's uncertainty principle.

We know of two objections to our analysis of this proposed experiment. The first one is that we have used classical mechanics in doing our calculations. But we note that, according to the usual interpretation of quantum mechanics, one always passes from quantum to classical mechanics in analysing an experiment (Heisenberg 1930). However, where the transition across the interface between quantum and classical mechanics occurs appears to be an unresolved problem (Peres and Rosen 1964, Maxwell 1972). But here this presents no real problem, since in the present case the motion of the wave packet is the same as that of the classical particle as shown by Ehrenfest's theorem (Schiff 1955). Furthermore, the experimental evidence of almost a century clearly shows that charged electrons do have the macroscopic trajectories assigned by classical mechanics.

A second objection is that, while we have shown that it may be possible to determine the simultaneous values of the position and momentum with a combined uncertainty $< \hbar/2$, this in itself does not constitute a violation of the 'true' meaning of Heisenberg's principle (Billette *et al* 1969, Ballentine 1969). According to these authors, Heisenberg's principle, as it should be properly understood, applies only to the 'future'. In our opinion, Heisenberg's principle, as it is actually used in the usual interpretation of quantum mechanics, clearly implies that—past, present and future—the position and momentum are 'smeared' to the extent that $\Delta x\ \Delta p \geqslant \hbar/2$ always. However, whatever the 'true' meaning of Heisenberg's principle may be, it does not affect the validity of the analysis carried out above, nor the conclusions summarised below. Therefore, a discussion of the various possible interpretations of Heisenberg's principle goes beyond the subject matter of this article.

We claim that we have shown that the usual interpretation predicts that, in general, $v_t \neq v_s$, but $\langle v_t \rangle = v_s$. Such a result would violate the pilot wave (and double-solution) interpretation. If, however, $v_t = v_s$, then it is possible to determine the simultaneous values of position and momentum such that $\Delta x\ \Delta p < \hbar/2$. Furthermore, if $v_t = v_s$, ψ is incomplete and $|\psi|^2$ is not a probability density under the suggested experimental conditions. These last results have also been derived from an analysis of α-particle emission (Robinson 1969b, 1978).

References

Andrade e Silva J 1967 *C. R. Acad. Sci., Paris* **264** 909
Ballentine L E 1969 *Can. J. Phys.* **47** 2417
Billette J J, Campillo D, Lee C, McConnell R D, Pariseau G and Fisher G 1969 *Can. J. Phys.* **47** 2415
Bohm D 1952a *Phys. Rev.* **85** 166
—— 1952b *Phys. Rev.* **85** 180
de Broglie L 1927 *J. Phys. Radium Serie VI* **8** 225
—— 1964 *The Current Interpretation of Wave Mechanics* (New York: Elsevier)
—— 1969 *C. R. Acad. Sci., Paris* **268** 277
Heisenberg W 1930 *Physical Principles of the Quantum Theory* (Chicago: University of Chicago Press)
Maxwell N 1972 *Am. J. Phys.* **40** 1431
Peres A and Rosen N 1964 *Ann. Phys., NY* **29** 366
Robinson M C 1969a *Can. J. Phys.* **47** 963
—— 1969b *Phys. Lett.* **30A** 69
—— 1978 *Phys. Lett.* **66A** 263
Schiff L I 1955 *Quantum Mechanics* (New York: McGraw-Hill) 2nd edn